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A METHOD TO PREDICT THE THERMAL PERFORMANCE OF PRINTED CIRCUIT BOARD MOUNTED SOLID STATE DEVICES

by

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A procedure has been formulated to predict the thermal performance of printed circuit board mounted solid state devices (specifically 14 and 16 pin DIP's and TO-3 and TO-66 transistor cases). The equivalent thermal network has been constructed for a single I.C. element mounted on a P.C. board. The network for the DIP case is slightly different from the network for the TO case. The thermal network is designed to account for all the possible heat flow paths from the element to the ultimate heat sink. The network is also

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20. (Continued). capable of accounting for the thermal interaction of a given element with other elements on the same board. This interaction is assumed to occur through heat conduction in the electrical connections between elements. Expressions for each of the thermal resistances in these networks has been formulated. This analytical model has been coded in the form of a digital computer program.

To verify the analytical procedure, an experimental P.C. board has been made. The board has 25 thick film resistors which are 14 pin DIP's. Each element on the board is instrumented with two thermocouples, one on the top and one on the bottom, to measure the average temperature of the element. Tests were conducted in the electronics cooling test facility of the Mechanical Engineering Department. Three different air flow rates and four different power settings to the board were used. Air inlet and exit temperatures were measured as well as the average temperatures of each element on the board.

The computer program which implements the analytical procedure has been run for several cases corresponding to the experimental runs. At the lower power levels the agreement with the experimental results is excellent. At higher power levels the analytical procedure predicts a higher average temperature than is observed experimentally.

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I. INTRODUCTION

The reliability of electronic systems is highly dependent on the ability to dissipate the relatively large amounts of Joulian heating that such systems can generate. If this heat is not removed efficiently overheating of critical components may occur. This of course could lead to ultimate system failure. The development of microminaturized electronic components, with their attendant extremely high volumetric heating rates, has made the problem of heat removal an even more critical one. Recently a great deal of interest has been shown in the investigation of the heat transfer problems associated with the cooling of electronic equipment [1-6]*.

When designing a heat removal system for electronic components, the packaging engineer is faced with many choices. These choices may range from the simplest natural convection cooled heat sinks to a very elaborate two phase heat exchanger system. In making his decision, the packaging engineer must consider many factors: the operating characteristics of the components themselves; the requirements of the electronic system designer with respect to component layout; the operating environment of the system. The packaging designer needs a technique which will allow him to predict the thermal performance of electronic system components.

1.1 OBJECTIVE

The objective of this work is to formulate a design procedure to be used in the prediction of the thermal performance of printed circuit board mounted solid state devices (specifically 14 and 16 pin DIP's and TO-3 and TO-66 transistor cases). The project consists of an analytical phase which

*Numbers in brackets indicate references listed in the Bibliography.

constitutes the actual formulation of the design procedure in the form of a digital computer program with appropriate documentation and an experimental phase which involves testing of actual P-C boards to verify the analytical model.

2. FORMULATION OF THE DESIGN PROCEDURE

The approach which has been adopted in the formulation of the design procedure is to construct an approximate thermal network [7] for each element on the printed circuit board. In this approach the element is considered to be a source of heat for which either the temperature or the rate of heat generation is known. An assessment is then made of each one of the possible paths by which this heat can be transferred to the ultimate sink. The thermal resistance for each one of these heat flow paths is formulated and these resistances are used to construct the thermal network for the element. The thermal resistance for a given heat flow path is defined such that:

$$Q = \frac{\Delta T}{R} \tag{1}$$

where

Q is the heat transfer rate (W) for a given heat flow path,

ΔT is the temperature difference (°C)

and R is the thermal resistance (°C/W) for the heat flow path.

This thermal network is then solved to give either the unknown element temperature or the unknown rate of heat generation.

The case styles of circuit elements being considered in this study were of two classes: 14 or 16 pin DIP's and TO-3 or TO-66 transistor cases. Schematic diagrams of these two types of elements as they are mounted on a P.C. board are shown in Figs. 1 and 2. The corresponding thermal network for each one is shown diagramatically in Fig. 3 and 4. The thermal networks for the two types of element differ ~~only~~ in the resistances which represent the heat flow paths from the element to the P.C. board. The DIP element is mounted so that the only conduction path from the element to the board is through the element leads (Fig. 1). The TO case is mounted with a mica insulator between the element and the P.C. board so that conduction heat transfer takes place from the base of the TO case through the mica to the P.C. board (Fig. 2). The following sections will describe the formulation of the individual resistances in the thermal network.

2.1 DIP ELEMENT RESISTANCES

As mentioned previously the thermal networks for the DIP and TO cases differ only in the resistances from the element to the board. For the DIP there are considered to be two primary heat flow paths and associated resistances from the element to the board. These are the heat flow path through the short element leads to the board and the heat flow path through the air gap between the element and the board. The thermal resistances associated with these heat flow paths are represented by $R_{L,S}$ and R_{GAP} respectively.

2.1-1 FORMULATION OF $R_{L,S}$:

Heat transfer in the short leads from the element to the board is assumed to occur by conduction only. This implies that the heat transfer rate is given by:

$$
\begin{aligned}
Q_{L,S} &= -kA \frac{dT}{dX} \\
&= k_L N_L A_C \frac{T_i - T_{Bi}}{L_L}
\end{aligned}
\tag{2}
$$

where

$Q_{L,S}$ = heat transfer rate through the short lead (W)

k_L = thermal conductivity of the lead (W/m/°C)

A_C = cross sectional area of the lead (m^2)

L_L = length of the lead (m)

T_i = temperature of the element (°C)

T_{Bi} = temperature of the board at point where the leads make contact (°C)

N_L = number of leads

The subscript i means the equation is written for the i^{th} element. With Eq. (2) for the heat transfer rate and the definition of R from Eq. (1), the resistance $R_{L,S}$ becomes:

$$R_{L,S} = \frac{L_L}{k_L A_C N_L} \tag{3}$$

2.1-2 FORMULATION OF R_{GAP}:

Since the gap between the element and the board is usually very small, it is assumed that to a good approximation there is no convective heat transfer but only conduction and radiation across the gap. With this assumption, the heat transfer rate across the gap can be written as:

$$Q_{GAP} = k_f A_G \frac{T_i - T_{Bi}}{\Delta x_G} + \sigma\left(\frac{\varepsilon}{2 - \varepsilon}\right) A_G \left(T_i^4 - T_{Bi}^4\right) \tag{4}$$

where

Q_{GAP} = heat transfer rate across the gap (watts)

k_f = thermal conductivity of air (watts/meter/°C)

A_G = surface area of the bottom of the element/(meters)^2

σ = Stefan-Boltzman constant

= 5.6697 x10^{-8} W/m/°K^4

ε = average emissivity of the element and the board

In formulating the radiation term in Eq. (4) the bottom of the element and the board are assumed to behave as two infinite parallel planes [7] whose emissivities are the same. This radiation term can be further simplified by linearizing. This can be done by expanding $(T_i^4 - T_{Bi}^4)$ in a

Taylor series and retaining only the linear portion. If this is done, the heat transfer rate can be rewritten as:

$$Q_{GAP} = \frac{k_f A_G}{\Delta x_G} (T_i - T_{Bi}) + \frac{4\sigma\varepsilon}{2 - \varepsilon} A_G T_i^3 (T_i - T_{Bi}) \tag{5}$$

Using the definition of thermal resistance from Eq. (1) this can be written as:

$$Q_{GAP} = \frac{T_i - T_{Bi}}{R_{G,C}} + \frac{T_i - T_{Bi}}{R_{G,R}} \tag{6a}$$

$$= \frac{T_i - T_{Bi}}{R_{GAP}} \tag{6b}$$

When $R_{G,C}$ and $R_{G,R}$ are the conduction and radiation thermal resistances across the gap. From the form of Eq. (6) it can be seen that these two resistances act in parallel and the total gap resistance can be written as:

$$R_{GAP} = \frac{R_{G,C} \ R_{G,R}}{R_{G,C} + R_{G,R}} \tag{7}$$

where

$$R_{G,C} = \frac{\Delta x_G}{k_f A_G} \tag{8a}$$

$$R_{G,R} = \frac{2 - \varepsilon}{4\sigma\varepsilon A_G T_i^3} \tag{8b}$$

It should be pointed out that $R_{G,R}$ and hence R_{GAP} is dependent on the element temperature T_i. This of course means that the network will be non-linear and an iterative technique will be used for solution.

2.2 TO ELEMENT RESISTANCES

For the TO case heat conduction through the mica insulator constitute the only resistance between the element and the board. Since the mica is a soft material and the contact pressures are high it is reasonable to neglect the contact resistance at the mica-element and mica-board interfaces. With this assumption, the heat transfer rate from the TO case to the board can be written as:

$$Q_{MICA} = k_M A_M \frac{T_i - T_{Bi}}{t_M} \tag{9}$$

The thermal resistance of the mica insulator then becomes:

$$R_M = \frac{t_M}{k_M A_M} \tag{10}$$

where:

t_M = thickness of the mica (meters)

k_M = thermal conductivity of the mica (W/M/°C)

A_M = surface area of the mica (meters^2)

2.3 FORMULATION OF THE EXTERNAL CASE RESISTANCE

The surface area of the element case will transfer heat by convection to the surrounding air which is at some specified temperature T_∞. The case can also transfer heat by radiation. The radiation is assumed to take place to a very large black body sink at some specified radiation sink temperature $T_{R\infty}$. The heat transfer rate from the element case can be written as:

$$Q_{CASE} = h_c A_{cA} (T_i - T_\infty) + \sigma \varepsilon_E A_{CA} (T_i^4 - T_{R\infty}^4) \tag{11}$$

By linearizing the radiation term this can be rewritten as:

$$Q_{CASE} = h_c A_{cA} (T_i - T_\infty) + 4\sigma\varepsilon_E A_{CA} T_{R\infty}^3 (T_i - T_{R\infty}) \tag{12}$$

Using the definition of thermal resistance from equation (1), the case resistance can be written as:

$$R_{CASE} = \frac{1}{h_c A_{CA} + 4\sigma\varepsilon_E A_R T_{R\infty}^3 \left[\frac{T_i - T_{R\infty}}{T_i - T_\infty}\right]} \tag{13}$$

where:

h_c = surface heat transfer coefficient ($W/m^2/°C$)

A_{CA} = element surface area (m^2)

A_R = radiation surface area (m^2)

ε_E = surface emissivity of the element

Since the nature of the flow over the element is highly dependent on the geometry of the element careful attention must be paid to the evaluation of the heat transfer coefficient. The geometry of the DIP is of course very different from the geometry of the TO case. This means that the evaluation of the heat transfer coefficient over each case style will have to be approached separately.

2.3-1 RESISTANCE OVER THE DIP CASE

The orientation of the DIP case with respect to the flow direction is an important factor in determining the heat transfer coefficient. For the purposes of this study the DIP case will be considered to be oriented in either one of two ways: long axis of the DIP parallel to the flow direction; or long axis of the DIP perpendicular to the flow direction. For either orientation the flow on the top and sides parallel to the flow is assumed to be laminar flat plate flow (Blasius flow). The flow on the sides perpendicular to the flow, the front and the back, is assumed to be stagnation flow.

In the case of flat plate flow the standard correlation for Blasius flow [8] can be used to obtain a heat transfer coefficient:

$$h_B = .664 \frac{k_f}{L} P_R^{1/3} Re_L^{1/2} \quad (14)$$

where

k_f = thermal conductivity of air $(\frac{W}{m°C})$

L = length of the flat plate under consideration (meters)

P_R = Prandtl number of air

Re_L = Reynolds number of the flow with respect to the length L

The Reynolds number is given by:

$$Re_L = \frac{V_f L}{\nu_f} \qquad (15)$$

where

V_f = the air velocity (m/sec)

ν_f = kinematic viscosity of air (m^2/sec)

The flow velocity can be obtained by considering the geometry of the array of printed circuit boards:

$$V_f = \frac{Q_f}{Z_B y_B} \qquad (16)$$

where

Q_f = volume rate of flow of air between adjacent boards (m^3/sec)

y_B = vertical height of the P.C. board (m)

Z_B = spacing between adjacent boards (m)

Combining all of these the heat transfer coefficient in Blasius flow can be written as:

$$h_B = .664 \frac{kf}{\nu_f^{1/2}} P_R^{1/3} \left[\frac{Q_f}{Z_B y_B}\right]^{1/2} \frac{1}{L^{1/2}} \qquad (17)$$

The value of the length L to be used will depend on which particular orientation is being considered.

For the case of stagnation flow the following correlation [8] can be used:

$$h_s = .57 \frac{k_f}{w} P_R^{0.4} Re_w^{1/2} \tag{18}$$

where

W = width of the surface perpendicular to the flow (meters)

Re_w = Reynolds number based on W

In a manner similar to the previous case of Blasius flow the heat transfer coefficient for stagnation flow can be written as:

$$h_s = .57 \frac{k_f}{\nu_f^{1/2}} P_R^{0.4} \left[\frac{Q_f}{Z_B Y_B}\right]^{1/2} \frac{1}{W^{1/2}} \tag{19}$$

The choice of lengths to be used to obtain the heat transfer coefficients from equations (17) or (19) as well as the choice of areas to be used with the respective heat transfer coefficients is dependent on the orientation of the element. For the DIP parallel to the flow $L = L_E$ in equation (17) and $W = W_E$ in equation (19). L_E and W_E are the length and width of the DIP element respectively. In this case the area associated with Blasius flow is assumed to consist of the top and bottom of the element, the long sides of the element plus the surface area of the short leads which experience the flow. The inclusion of the area of the short leads is to compensate for the fact these leads in reality behave as fins and transfer heat from their surface to the environment. In formulating the resistance for these leads, the fin effect was neglected and only conduction along the length of the lead was considered. The inclusion of the lead surface area in the evaluation of the DIP case resistance compensates for neglecting the fin effect. The surface area for stagnation flow is assumed to consist of the short sides which are perpendicular to the flow. With this in mind, the convective conductance hA can be written as:

$$hA = h_{B_{Le}} [2 L_eW_e + L_et_e + 2 N_LL_LW_L] + t_eh_{s_{we}} [2 W_et_e] \tag{20}$$

where

$h_{B_{Le}}$ = Blasius heat transfer coefficient with $L = L_e$

$h_{s_{we}}$ = stagnation flow heat transfer coefficient with $W = W_e$

t_e = thickness of the DIP case (m)

For the DIP perpendicular to the flow similar reasoning leads to the following expression for the convective conductance:

$$hA = h_{B_{We}} [L_eW_e + 2 W_et_e] + t_eh_{S_{Le}} [L_et_e + 2 N_LL_LW_L] \tag{21}$$

where

$h_{B_{We}}$ = Blasius heat transfer coefficient with $L = W_e$

$h_{S_{Le}}$ = stagnation flow heat transfer coefficient with $W = L_e$

The area to be used in the radiation term is considered to be the surface of the element and short leads which can be "seen" by the radiation sink. This is given as:

$$A_R = L_eW_e + 2 W_et_e + L_et_e + N_LL_LW_L \tag{22}$$

These expressions for heat transfer coefficient and area can now be used in equation (13) to obtain the appropriate form for R_{CASE} corresponding to the element orientation under consideration.

2.3-2 RESISTANCE OVER THE TO CASE

Because of the generally circular shape of the TO case, the flow pattern and hence the heat transfer coefficient is assumed to be independent of orientation. For the TO case the flow over the top is assumed to be Blasius flat plate flow while the flow around the sides is assumed to correspond to flow over a cylinder. For the flow over the top the heat transfer coefficient can be obtained by using equation (17) with

$$L = L_D = \frac{\sqrt{\pi}}{2} D \tag{23}$$

where

D = diameter of the cylindrical TO case

For the heat transfer coefficient over the cylindrical surface a modified Hilpert correlation as given on page 260 of McAdams [9] can be used:

$$h_{cyL} = \frac{k_f}{D} \left[0.32 + 0.43\, Re_D^{1/2}\right] \tag{24}$$

where

Re_D = Reynolds number based on the diameter D

Using the definition of the Reynolds number and the expression for the velocity from equation (16) the cylindrical heat transfer coefficient can be written as:

$$h_{cyL} = \frac{k_f}{D} \left[0.32 + \frac{0.43}{\nu_f^{1/2}} \left(\frac{Q_f}{Z_B Y_B}\right)^{1/2} D^{1/2}\right] \tag{25}$$

The convective conductance for the TO case can now be written as:

$$hA = h_{B_{LD}} \frac{\pi}{4} D^2 + h_{cyL} \pi DH \tag{26}$$

where

H = height of the cylindrical TO case (meters)

The area to be used in the radiation term is simply:

$$A_R = \frac{\pi}{4} D^2 + \pi DH \tag{27}$$

These expressions can now be used in equation (13) to obtain R_{CASE} for the TO case.

The choice of the proper expressions for the heat transfer coefficient as well as the choice of the appropriate areas to use is based on a great deal of intuition gained through previous experience [1]. These terms can be changed or adjusted as more experience with the model is gained.

2.4 FORMULATION OF BOARD RESISTANCE

The portion of the printed circuit board which lies directly under the element is assumed to be at a uniform temperature T_{Bi}. The back of this portion of the board (the side facing away from the element, see Fig. 5) exchanges heat directly with the environment by convection and radiation. By linearizing the radiation the heat transfer rate from this portion of the board can be written as:

$$Q_{BB} = h_{BB} A_{BB} (T_{B_i} - T_\infty) + 4\sigma \, \varepsilon_B A_{BB} T_{R_\infty}^{\;3} (T_{B_i} - T_{R_\infty}) \tag{28}$$

where

h_{BB} = heat transfer coefficient over the back of the board ($W/m^2/°C$)

A_{BB} = area of the back of the board directly under the element (m^2)

T_{B_i} = temperature of the back of the board directly under element i (°C)

ε_B = emissivity of the board

from this the thermal resistance can be written as:

$$R_{BB} = \frac{1}{h_{BB}A_{BB} + 4\sigma\varepsilon_B A_B T_R{}^3 \left(\frac{T_{B_i} - T_R}{T_{B_i} - T_\infty}\right)} \tag{29}$$

If the back of the board is assumed to be a flat plate the heat transfer coefficient h_{BB} can be found by using equation (17) with $L = X_B$, where X_B is the length of the board in the flow direction given in meters.

2.5 FORMULATION OF BOARD FIN RESISTANCE

The major portion of the board can be considered to behave as a circumferential fin with base temperature T_{Bi}. The fin loses heat from its surface by convection and radiation to the environment. For the purpose of calculating a heat transfer rate equivalent inner and outer radius must be established. The inner radius (see Fig. 5) is defined as the radius of a circle with area equal to the area of the planform of the I.C. element (A_{BB}). This gives

$$L_1 = \sqrt{\frac{1}{\pi} A_{BB}} \tag{30}$$

where L_1 is the inner radius. The outer radius is defined as the radius of a circle with area equal to the fraction of the total board area occupied by a single element. This average area is given by

$$A_{AV} = \frac{X_B Y_B}{N_e} \tag{31}$$

where N_e is the total number of elements on the board. From this the outer radius can be written as:

$$L_2 = \sqrt{\frac{X_B Y_B}{\pi N_e}} \tag{32}$$

For a circumferential fin with convection and linearized radiation from the surface standard fin analysis gives the following governing differential equation for the temperature distribution:

$$\frac{1}{x} \frac{d}{dx} \left(x \frac{d\theta}{dx}\right) - \left(\frac{2h_B}{kt} + \frac{8\sigma\varepsilon_B T_{R\infty}^3}{kt}\right) \theta = \frac{8\sigma\varepsilon_B T_{R\infty}^3}{kt}(T_\infty - T_{R_\infty}) \tag{33}$$

where

$$\theta = T - T_\infty$$

$$t = \text{thickness of the fin}$$

The boundary conditions for this equation are:

$$\begin{aligned} x &= L_i \qquad T = T_{B_i} \\ x &= L_2 \qquad \frac{dt}{dx} = 0 \end{aligned} \tag{34}$$

The solution to equation (33) can be found in Arpaci [10] page 153. Using the boundary conditions (34) to obtain the constants of integration, Fouriers law will yield the heat transfer rate through the base of the fin:

$$Q_{BF} = -2\pi k L_1 t m^2 \left[D_1 I_1(mL_1) - D_2 K_1(mL_1)\right] \tag{35}$$

where

$$m^2 = \frac{2h_B}{kt} + \frac{8\sigma\varepsilon_B T_{R_\infty}}{kt} \tag{36}$$

$$= m_c^2 + m_R^2$$

$$D_1 = \frac{\left[T_{B_i} - T_\infty \quad \frac{m_R^2}{m^2} (T_\infty - T_{R_\infty})\right] K_1(mL_2)}{I_o(mL_1) K_1(mL_2) + K_o(mL_1) I_1(mL_2)} \tag{37}$$

$$D_2 = \frac{[T_{B_i} - T_\infty + \frac{m_R^2}{m^2} (T_\infty - T_{R\infty})] \; I_1(mL_2)}{I_o(mL_1)\; K_1(mL_2) + K_o(mL_1)\; I_1(mL_2)} \tag{38}$$

and I and K are modified Bessel functions. From equation (35) the thermal resistance can be obtained:

$$R_{BF} = \frac{f(m\; L_1\; L_2)}{2\; ktL_1 m\; [1 + \frac{m_R^2}{m^2} (\frac{T_\infty - T_{R\infty}}{T_o - T_\infty})]} \tag{39}$$

where the function f is given by:

$$f = - \frac{I_o(mL_1)\; K_1(mL_2) + K_0(mL_1)\; I_1(mL_2)}{I_1(mL_1)\; K_1(mL_2) + K_1(mL_1)\; I_1(mL_2)} \tag{40}$$

2.6 RESISTANCES IN THE LEADS BETWEEN ELEMENTS

The leads which connect one element with another can act as conduction heat transfer paths. Through these interconnecting leads it is possible for the temperature of one element to influence the temperature of several others. Some of the leads from a particular element may be so long that they behave as infinite fins and do not represent an interaction path. Both cases can be treated by the same method of analysis.

2.6-1 FORMULATION OF R_{Lij}

The lead can be considered to be a fin of constant cross section with temperature T_i (corresponding to element i) at one end and temperature T_j (corresponding to element j) at the other end. The temperature distribution in such a fin with convection and linearized radiation on the surface is given by the following differential equation and boundary conditions:

$$\frac{d^2T}{dx^2} - \frac{h(2t_L + W)}{kt_L W}(T - T_\infty) - \frac{4\sigma\varepsilon T_{R\infty}^3(2t_L + W)}{kt_L W}(T - T_{R_\infty}) = 0$$

$$X = 0 \qquad T = T_i \tag{41}$$

$$X = L \qquad T = T_j$$

where

t_L = lead thickness

W = lead width

The solution to this equation can be found in Arpaci [10]. From this solution the heat transfer at x = 0 can be evaluated:

$$Q_o = kt_L Wm\ [(\theta_i + \theta_R)\ \coth mL_{ij} - \frac{\theta_i + \theta_R}{\text{sink } mL_{ij}} \tag{42}$$

where

$$m^2 = [\frac{h(2t_L + W)}{kt_L W}] + [\frac{4\sigma\varepsilon_B\ T_{R\infty}^3\ (2t_L + W)}{kt_L W}]$$

$$= m_C^2 + m_R^2 \tag{43}$$

$$\theta_i = T_i - T_\infty$$

$$\theta_j = T_j - T_\infty$$

$$\theta_R = \frac{m_R^2}{m^2}(T_\infty - T_{R_\infty})$$

From the heat transfer rate the thermal resistance can be obtained:

$$R_{L_{ij}} = \frac{\theta_i}{kt_L Wm\ [(\theta_i + \theta_R)\ \coth mL_{ij} - \frac{\theta_j + \theta_R}{\sinh mL_{ij}}]} \tag{44}$$

The length L_{ij} can be taken as the rectangular distance from element i to element j (see Fig. 6) given as:

$$L_{ij} = \sqrt{(X_i - X_j)^2} + \sqrt{(y_i - y_j)^2} \tag{45}$$

where x_i, x_j, y_i and y_j give the location of elements i and j on the P.C. board.

2.6-2 FORMULATION OF R_{L_∞}

If the lead is long enough the effect of the temperature T_j will disappear and the lead will behave as if it were an infinitely long fin. The same behavior can be attributed to any of the element leads which connect to the pins on the bottom of the P.C. board. The thermal resistance $R_{L\infty}$, for such leads can be obtained from equation (44) for R_{Lij} by taking the limit as L_{ij} becomes very large. This gives:

$$R_{L_\infty} = \frac{\theta_i}{kt_L Wm\ (\theta_i + \theta_R)} \tag{46}$$

2.7 SOLUTION OF THE THERMAL NETWORKS

The thermal network equations for each element type can be easily derived by writing simple energy balances at the two nodes i and Bi. Since the networks for the DIP and TO cases are slightly different the network equations will be different for the two element types.

2.7-1 NETWORK EQUATIONS FOR THE DIP ELEMENT

For the DIP case the energy balance for node i, the element itself is:

$$Q_{TOT} = Q_{CASE} + Q_{GAP} + Q_{L,S} \tag{47}$$

$$= \frac{T_i - T_\infty}{R_{CASE}} + \frac{T_i - T_{Bi}}{R_{GAP}} + \frac{T_i - T_{Bi}}{R_{L,S}}$$

$$= \frac{T_i - T_\infty}{R_{CASE}} + \left(\frac{1}{R_{GAP}} + \frac{1}{R_{L,S}}\right)(T_i - T_{Bi}) \qquad (48)$$

where Q_{TOT} is the total heat transfer rate from the element under consideration. This is the rate of energy generation in the element by Joulian heating. It is this quantity which must be dissipated to prevent overheating. At node B_i the energy balance gives:

$$Q_{GAP} + Q_{L,S} = Q_{BB} + Q_{BF} + Q_{Lij} + Q_{L\infty} \qquad (49)$$

Substituting for the heat transfer rates in terms of the thermal resistances gives:

$$(T_i - T_{Bi})\left(\frac{1}{R_{GAP}} + \frac{1}{R_{L,S}}\right) = \frac{T_{Bi} - T_\infty}{R_{BB}} + \frac{T_{Bi} - T_\infty}{R_{BF}} + (T_{Bi} - T_\infty)\sum_j \frac{1}{R_{Lji}} + \frac{T_{Bi} - T_\infty}{R_{L\infty}} \qquad (50)$$

$$= (T_{Bi} - T_\infty)\left[\frac{1}{R_{BB}} + \frac{1}{R_{BF}} + \sum_j \frac{1}{R_{Lij}} + \frac{1}{R_{L\infty}}\right] \qquad (51)$$

The summation on the right hand side of equation (51) is made over all other elements j which have leads connecting them to element i. Equations (48) and (51) are two simultaneous algebraic equations for either T_i and T_{Bi} given Q_{TOT} or for Q_{TOT} and T_{Bi} given T_i.

2.7-2 NETWORK EQUATIONS FOR THE TO CASE

The energy balance for the TO case, node i, yields:

$$Q_{TOT} = Q_{CASE} + Q_{MICA} + Q_{Lij} + Q_{L\infty} \tag{52}$$

$$= \frac{T_i - T_\infty}{R_{CASE}} + \frac{T_i - T_{Bi}}{R_{MICA}} + (T_i - T_\infty)\sum_j \frac{1}{R_{Lij}} + \frac{T_i - T_\infty}{R_{L\infty}}$$

$$= (T_i - T_\infty)\left[\frac{1}{R_{CASE}} + \sum_j \frac{1}{R_{Lij}} + \frac{1}{R_{L\infty}}\right] + \frac{(T_i - T_{Bi})}{R_{MICA}} \tag{53}$$

At node B_i the energy balance gives:

$$Q_{MICA} = Q_{BF} + Q_{BB} \tag{54}$$

$$\frac{T_i - T_{Bi}}{R_{MICA}} = (T_{Bi} - T_\infty)\left[\frac{1}{R_{BF}} + \frac{1}{R_{BB}}\right] \tag{55}$$

Equations (53) and (55) are the network equations for the TO case. They can be used to obtain either T_i and T_{Bi} given Q_{TOT} or Q_{TOT} and T_{Bi} given T_i.

2.8 SOLUTION PROCEDURE

It should be pointed out that some of the thermal resistances which make up the coefficients in the equation sets (48) and (51) or (53) and (55) depend on the temperatures T_i and T_{Bi}. This of course means that the equations are nonlinear and must be solved by an iterative procedure. The procedure is carried out by first guessing an initial set of the unknown temperatures. These are used to calculate the thermal resistances which are then used to solve the network equations for a new set of unknown temperatures. These are compared to the previous set of temperatures. If the temperatures from two successive iterations differ from each other by only some small amount the procedure is assumed to have converged. If successive iterations do not agree, the procedure is continued until convergence is achieved. The procedure has been coded in Fortran IV for implementation on a digital computer. A copy of the computer program is contained in Appendix A.

3. EXPERIMENTAL APPROACH

A series of experiments have been carried out for the purpose of verifying the analytical procedure. The experimental apparatus is the same as that used by Marto and Kelleher [1]. Figure 7 is a photograph of the complete experimental set-up. A minor modification was made to the test section which consisted of removing the cylindrical spacers between card guides. These spacers had been installed so that the test section would simulate the air transport rack. For these test this simulation was not desired and removing the spacers provided a more uniform air flow through the test section.

The experiments were conducted with a fiberglass epoxy P. C. board. The board had 25 thick film resistors which were 14 pin DIPs. The resistors were mounted in five rows of five elements each with the long axis of the elements aligned with the flow direction. Each element on the board is instrumented with two thermocouples, one on the top and one on the bottom of the element. Each pair of thermocouple is connected in parallel so that the output gives the average of the top and bottom temperatures for each element. Figure 8 shows photographs of the front and back of the P. C. board. As can be seen in this figure, all thermocouple leads were passed through small holes drilled near each element and run down the back of the board. This was done so as to disturb the flow over the elements themselves as little as possible.

Tests were conducted using the nozzle inlet described in [1]. Three different air flow rates and four different power settings to the board were used. Air inlet and exit temperatures were measured as well as the average temperatures of each element on the board. The test board was placed in the middle of the three card guides in the test section. Dummy boards were placed in the card guides on either side of the test board. The dummy boards were instrumented with thermocouples and their average temperature was measured during each run.

4. RESULTS AND CONCLUSIONS

The computer program which implements the analytical procedure presented in Chapter 2 has been run for several cases corresponding to the experimental runs. A P. C. board having 25 elements, all 14 pin DIPs, was assumed. The elements were in five rows of five elements with each element aligned with the flow direction. A value for the total power dissipated by the elements on the board was assumed and the program was run for a series of flow rates. This gave the element temperatures corresponding to each flow rate. This procedure was done for the four power settings corresponding to those of the experiment. The net result is shown in Figs. 7 and 8 where the curves represent the output of the computer program and the circles are the corresponding experimental points. Figure 7 shows the average element temperature as a function of flow rate for 4, 9, 15 and 22 watts per board. At the lower power levels the agreement with the experimental results is excellent. At higher power levels the analytical procedure predicts a higher average temperature than is observed experimentally. From a designers point of view this leads to a conservative design which is desirable.

Figure 8 shows the maximum element temperature as a function of flow rate for the four power settings. The agreement with the experiment is still good although the analytical procedure has a tendency to predict lower maximum temperatures than are experimentally observed.

The discrepancy in the maximum temperature is believed to occur from the fact that the radiation sink temperature was assumed to be the same as the ambient air temperature. This was done because it was assumed that a designer would have no other information for this case a *priori*. The result of the lower radiation sink temperature is to allow a greater amount of radiation heat transfer especially at the higher power settings (higher temperatures). This of course would lead to a lower average element temperature.

4.1 RECOMMENDATIONS FOR FUTURE WORK

The analytical procedure presented in this report should be viewed as a first attempt to model an extremely complex heat transfer problem. All three modes of heat transfer are present, the geometry and hence the flow patterns are extremely complex. Simple models have been used to analyze the heat transfer processes and many simplifying assumptions have been made to make the problem tractable. The result is an approximate procedure which seems to work. It should be remembered that the procedure has been verified in only one very simple situation. It is strongly recommended that a thorough experimental program be carried out to assure the validity of the procedure for cases other than those used in the experimental program.

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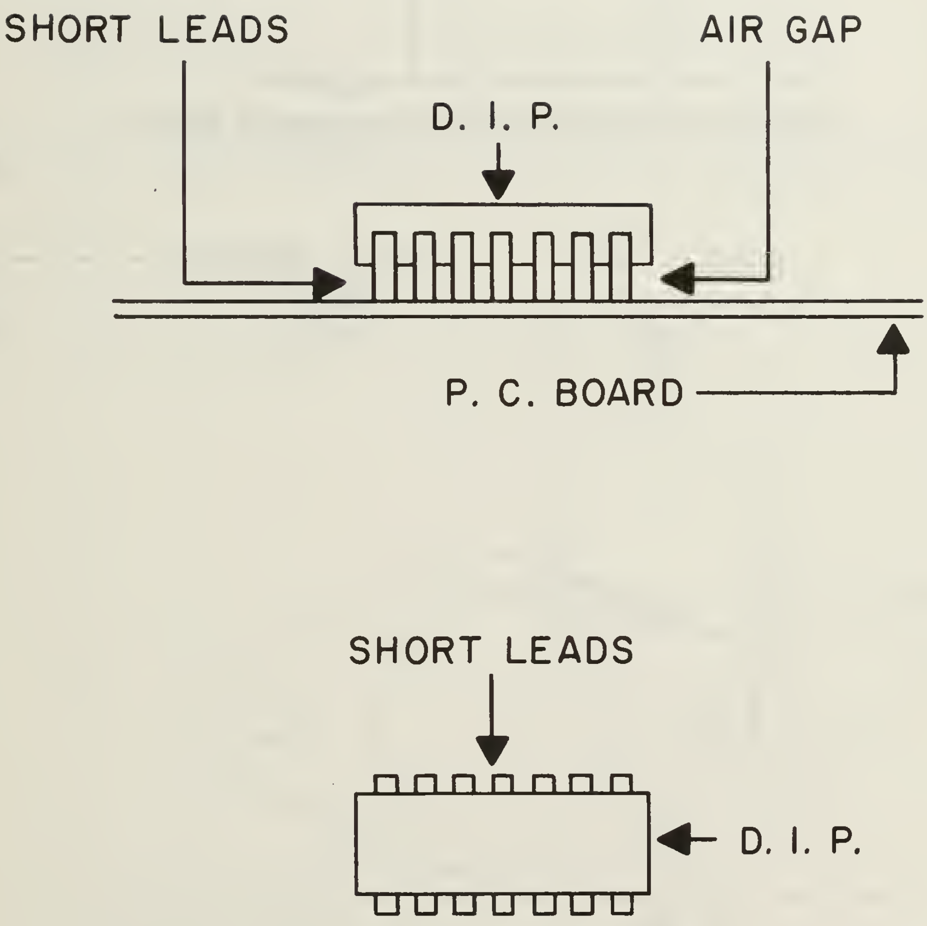
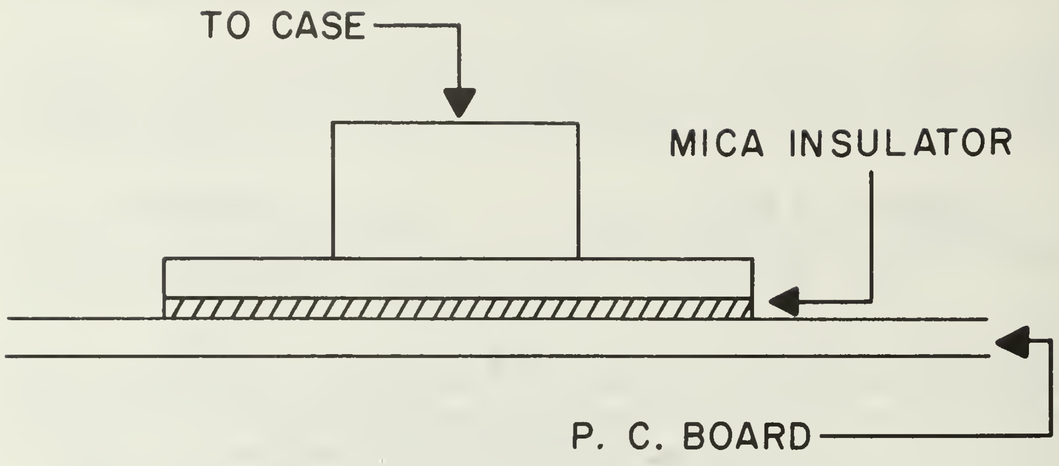


Fig. 1 Schematic Diagram of 14 Pin DIP



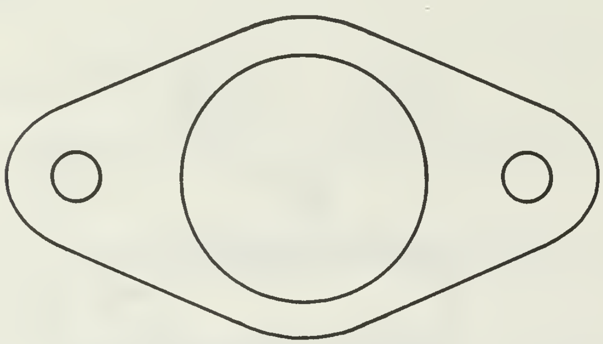


Fig. 2 Schematic Diagram of TO Case

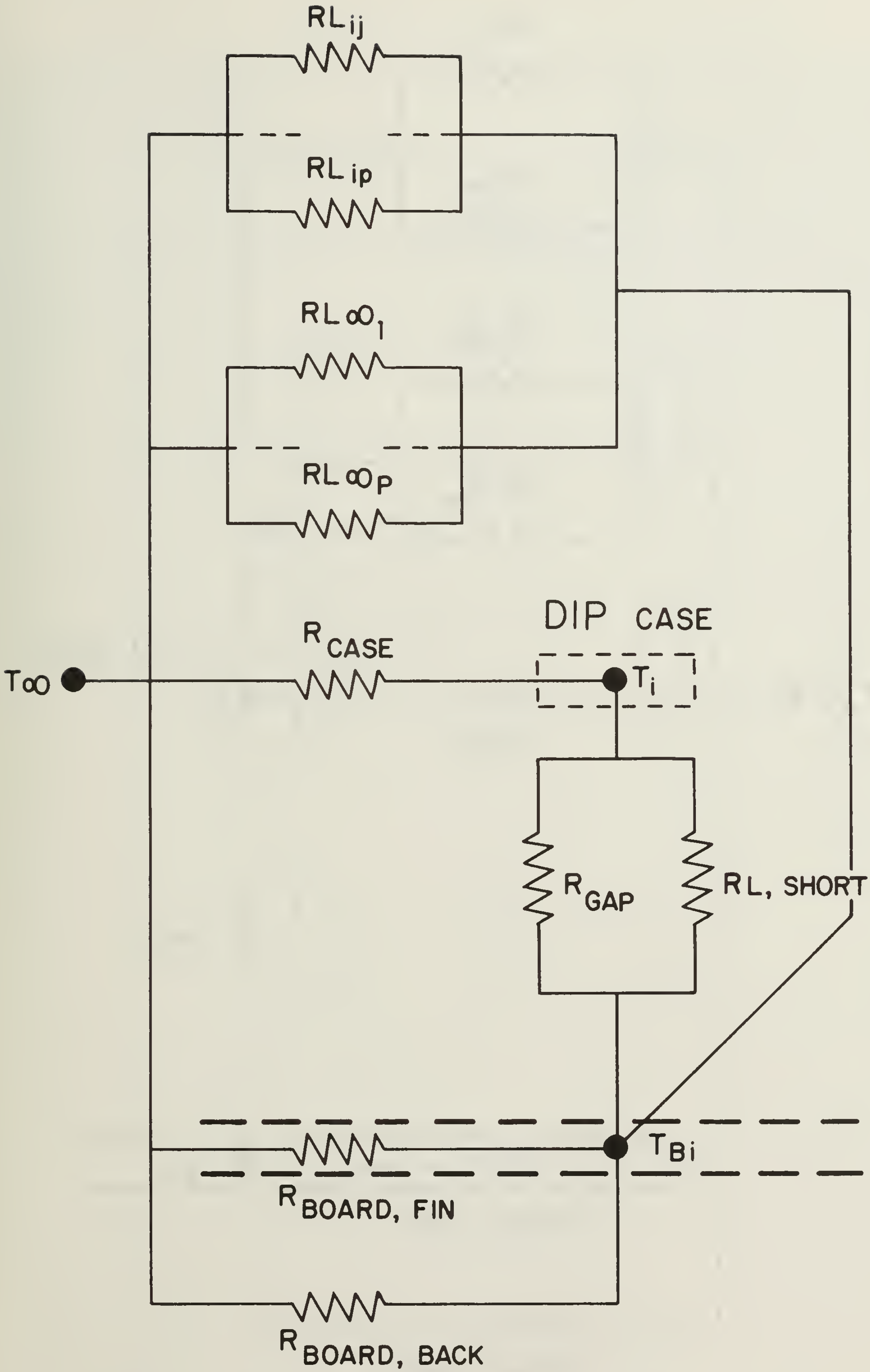


Fig. 3 Thermal Network for DIP

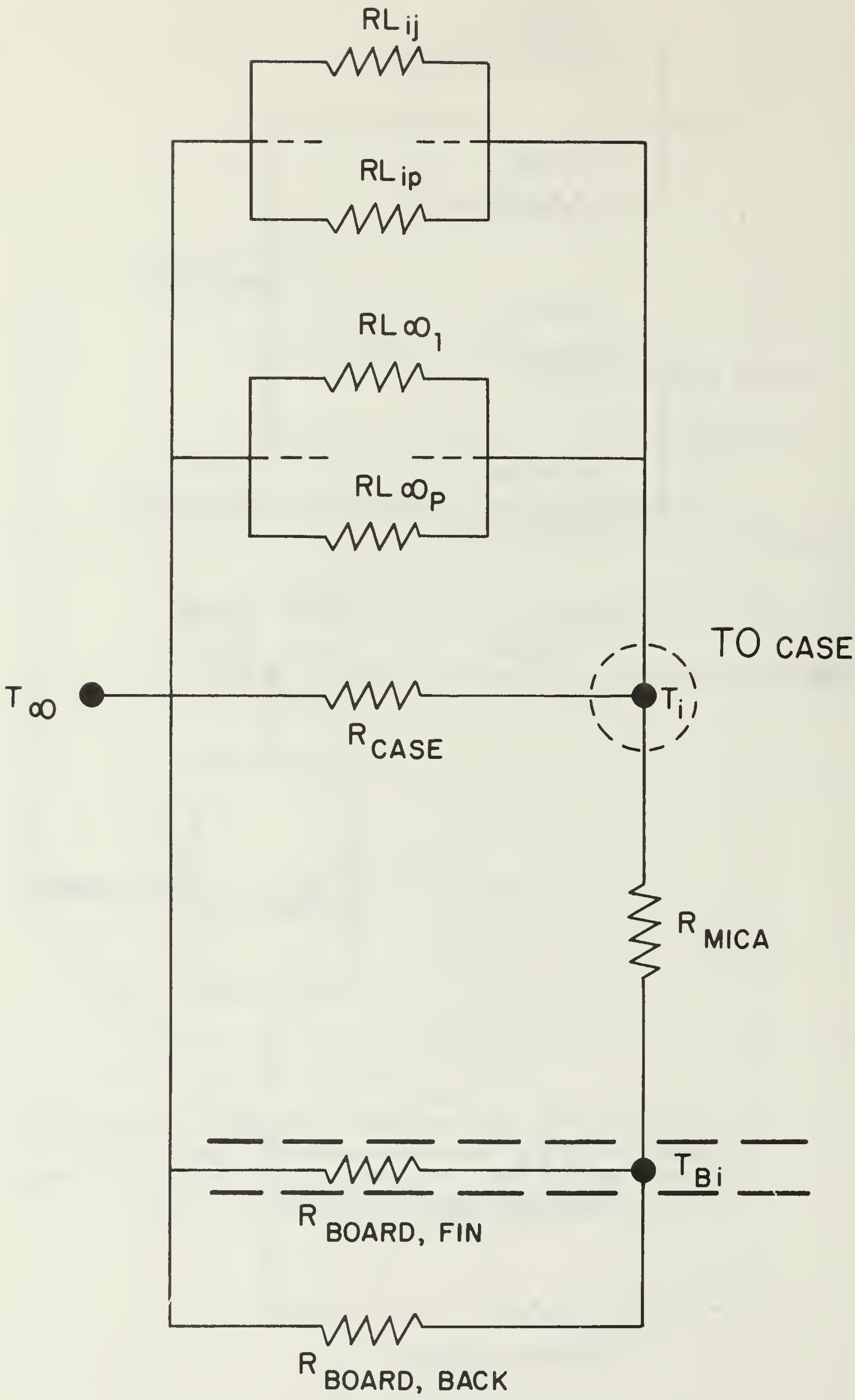


Fig. 4 Thermal Network for TO Case

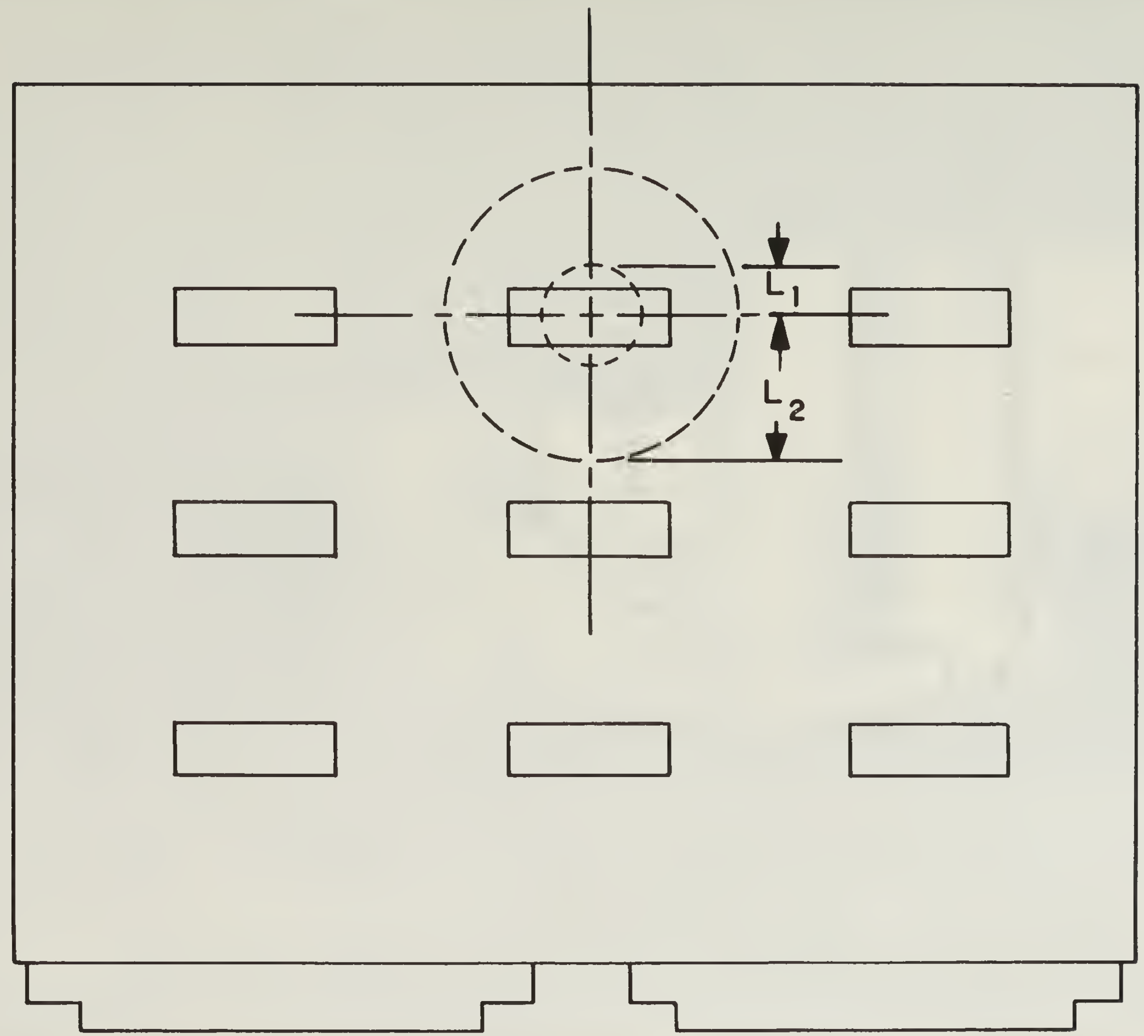


Fig. 5 Equivalent Dimensions for Board Fin Resistance

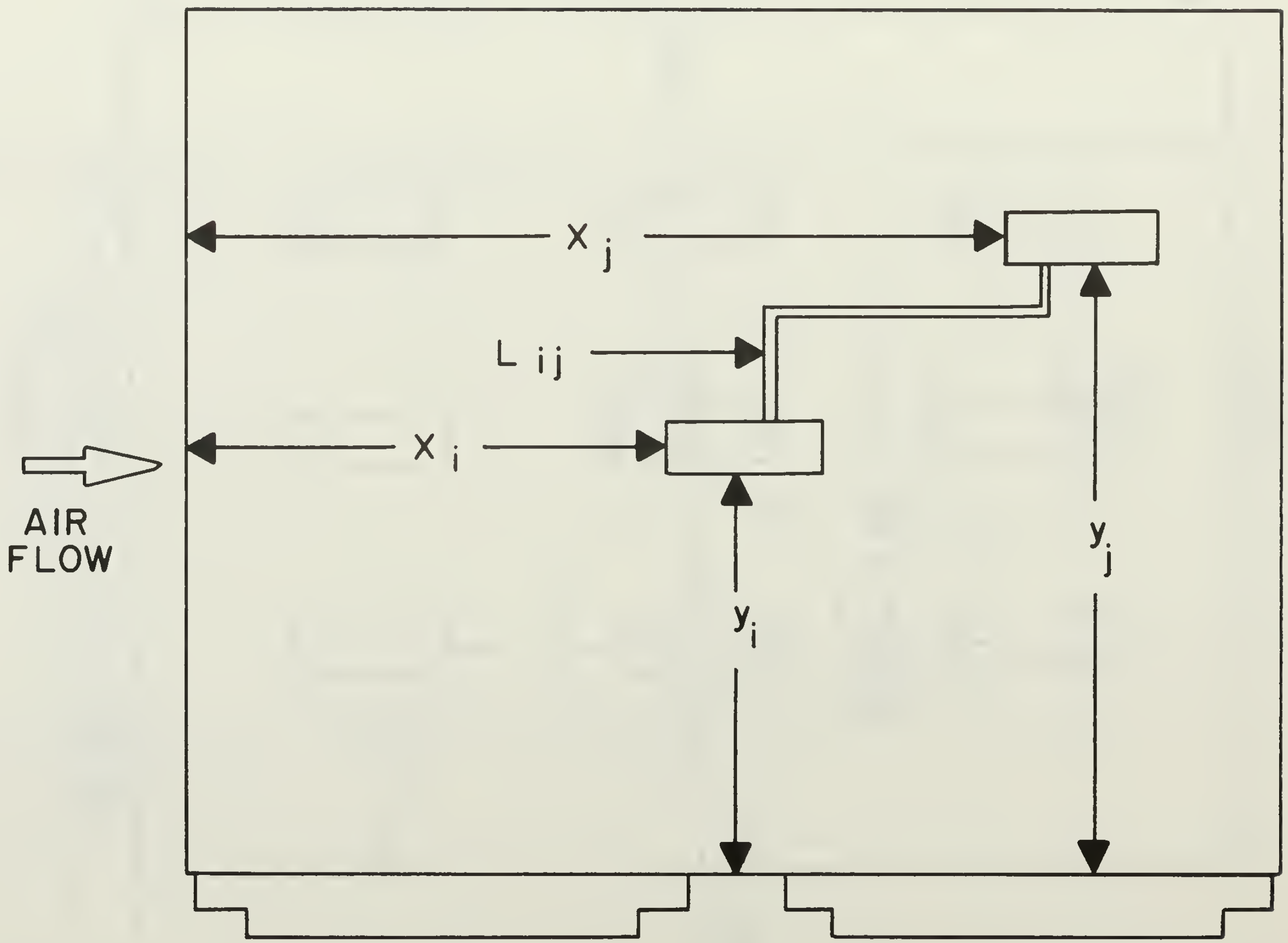


Fig. 6 Equivalent Dimensions for Lead Resistance

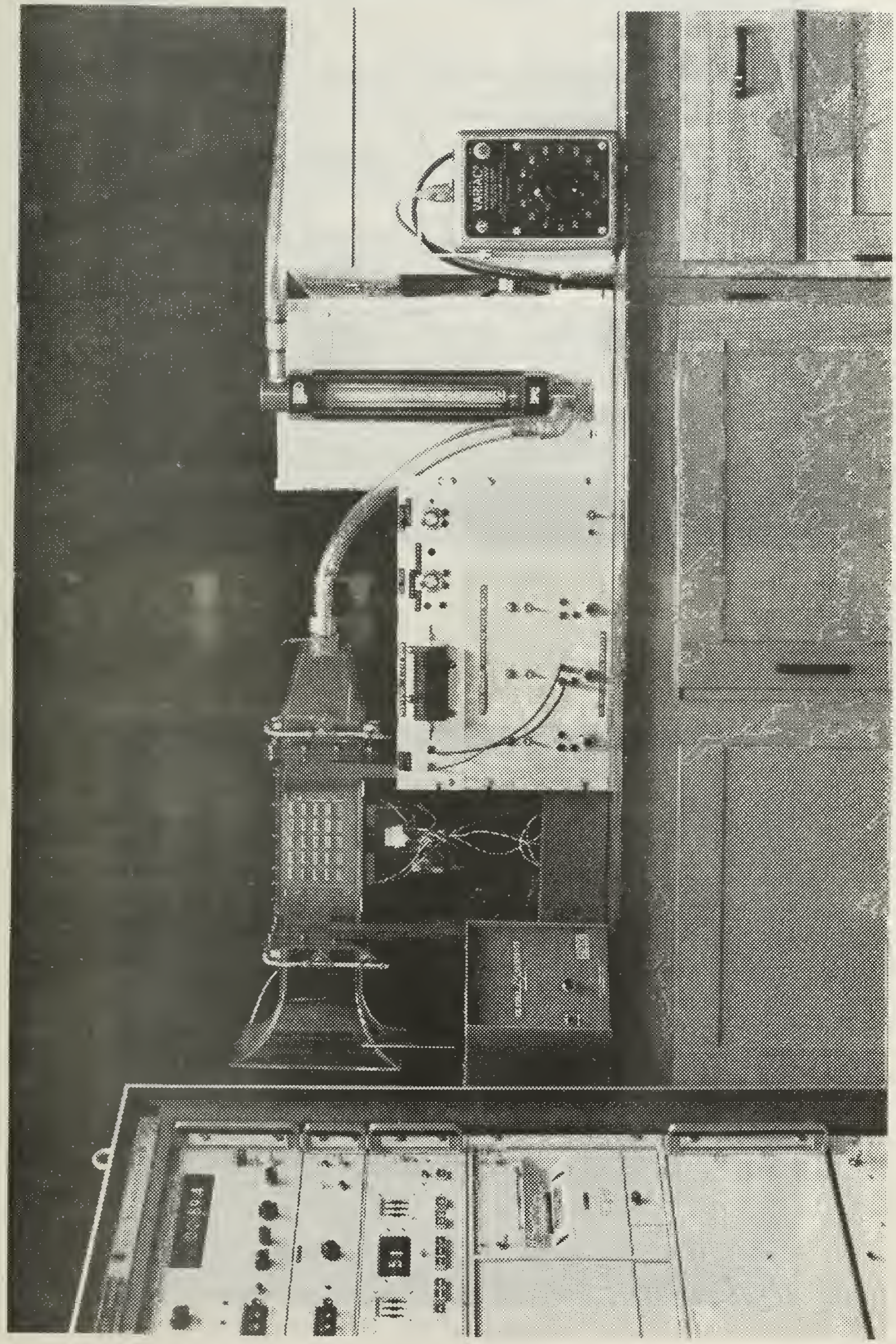
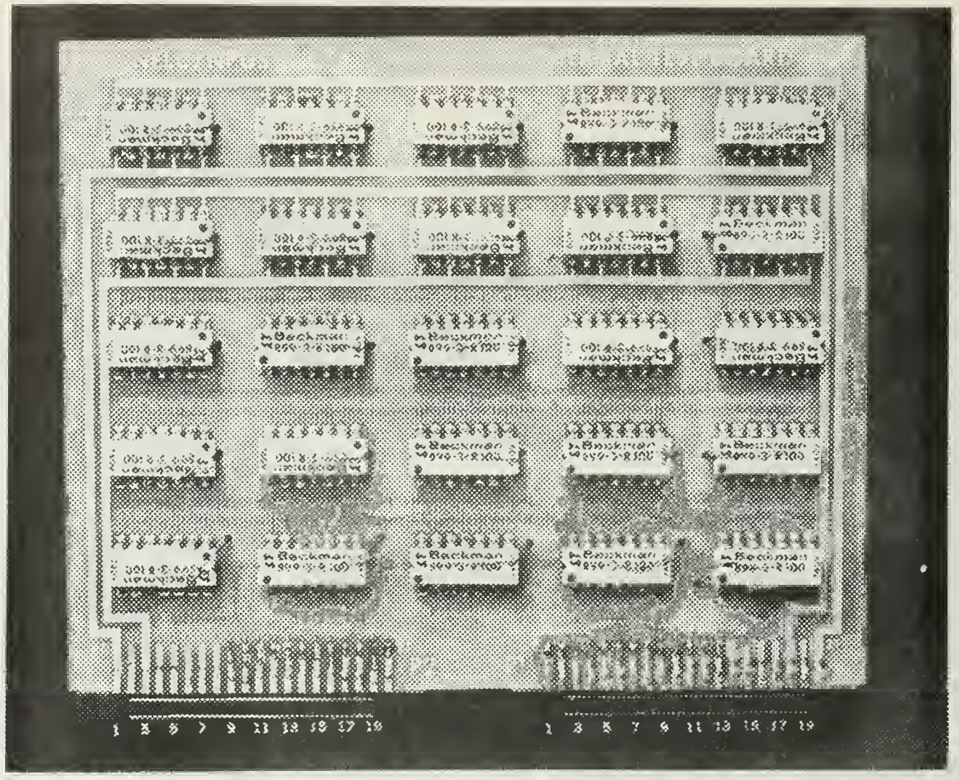
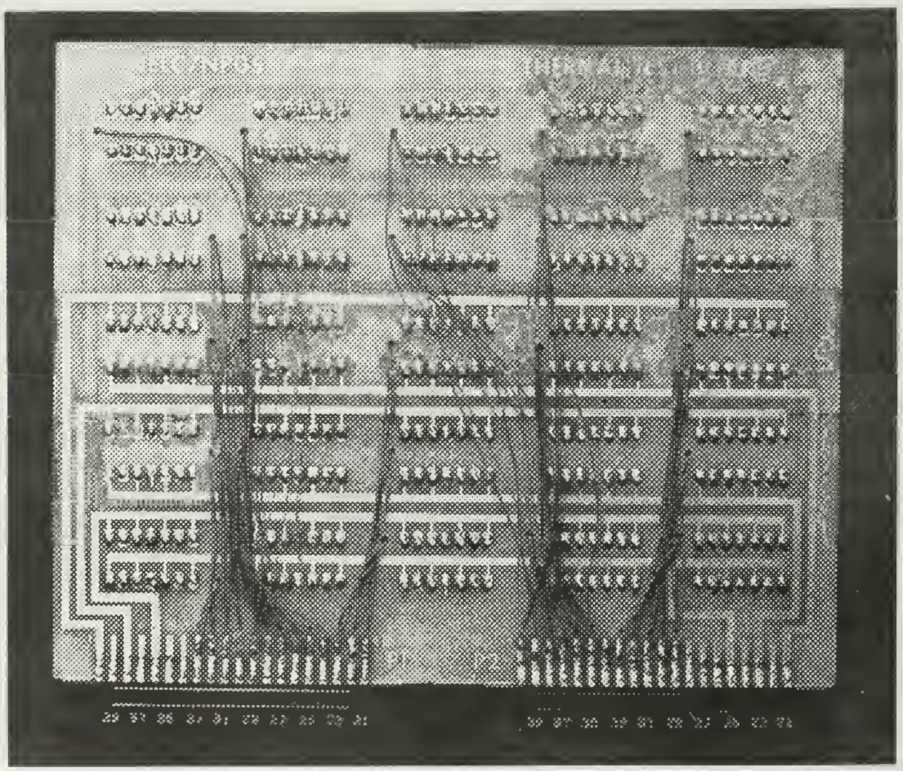


FIG. 7 EXPERIMENTAL SET-UP



FRONT OF BOARD



BACK OF BOARD

FIG. 8

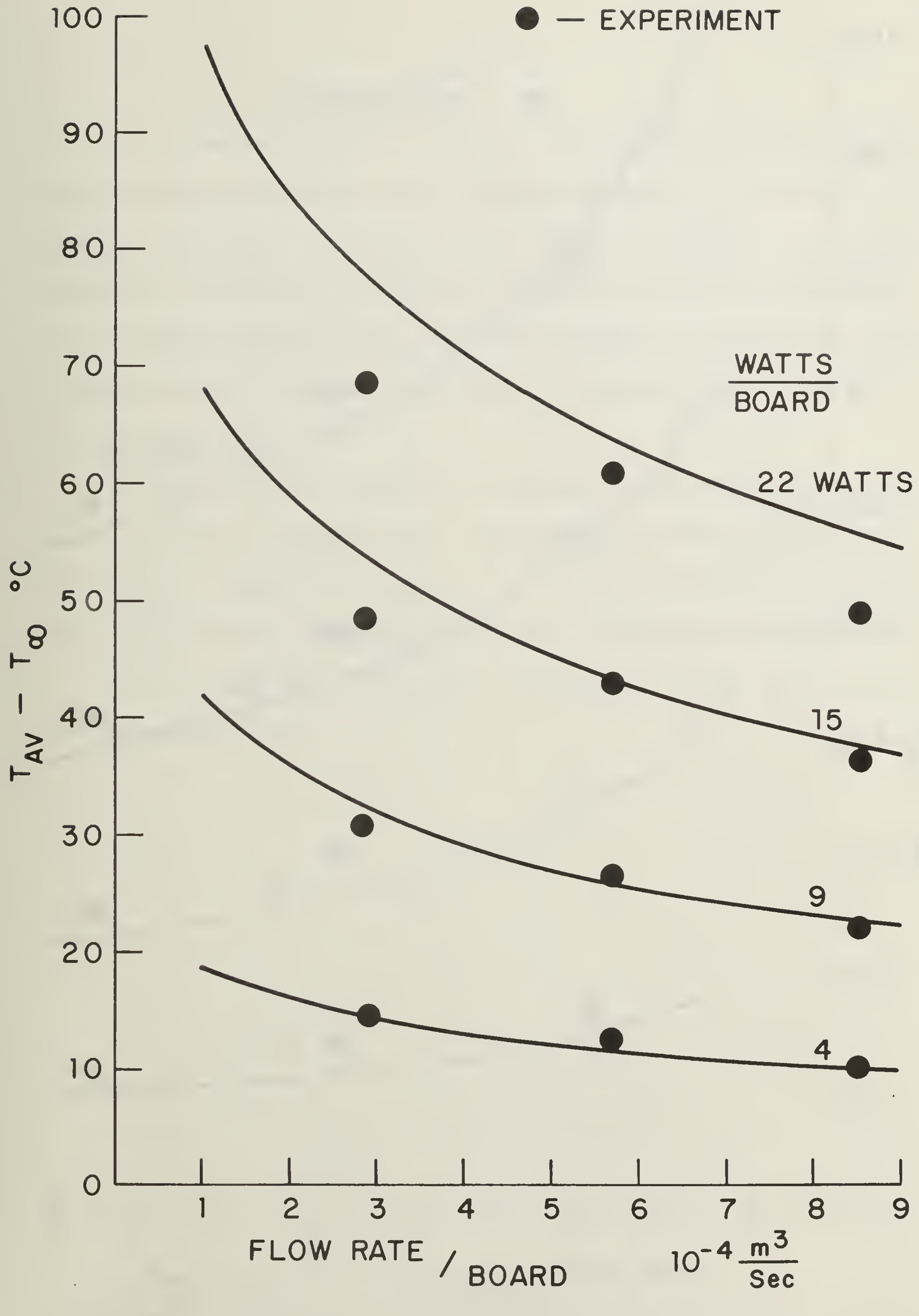
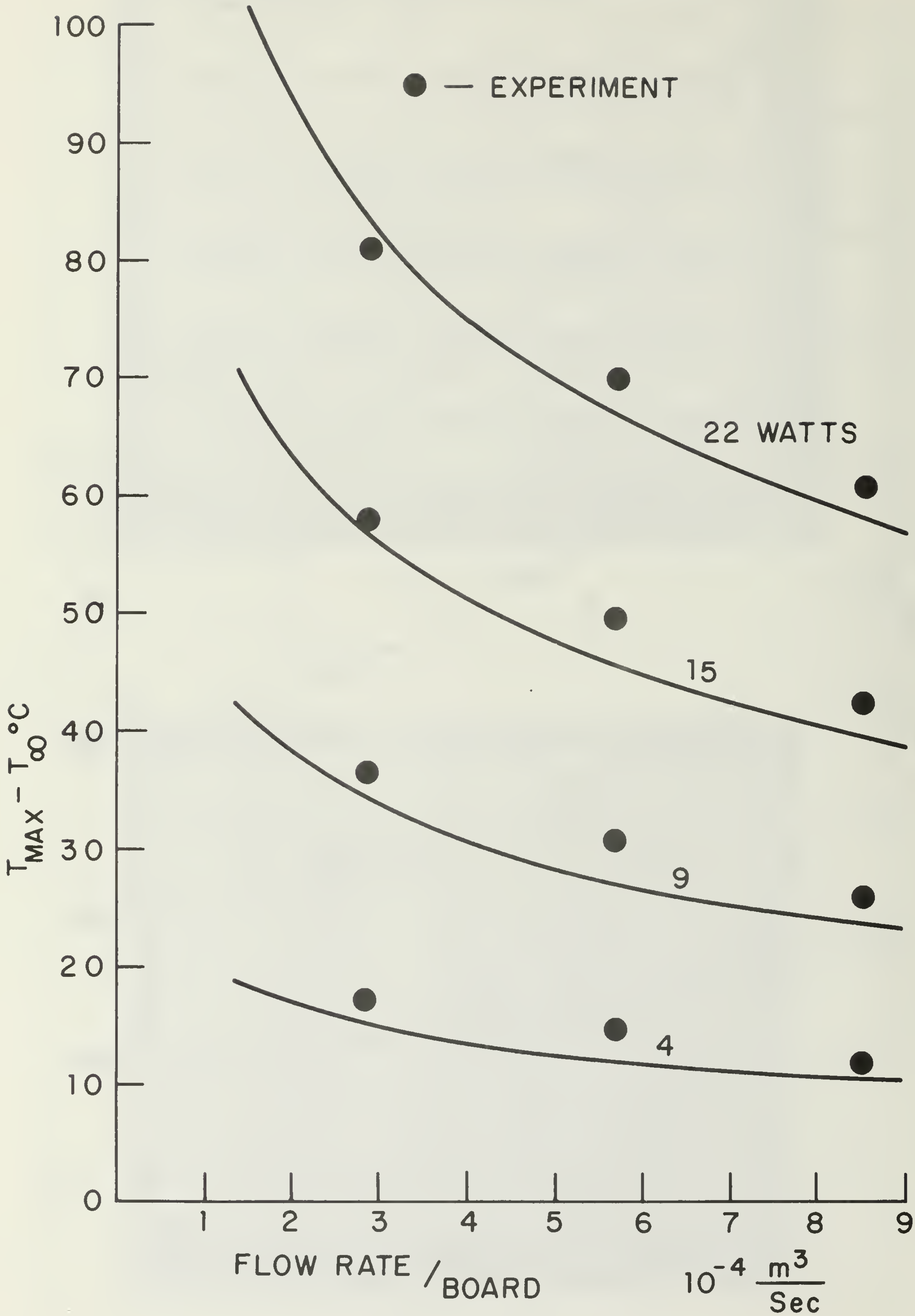

— EXPERIMENT
WATTS / BOARD
22 WATTS
15
9
4
$T_{AV} - T_{\infty}$ °C
100
90
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
FLOW RATE / BOARD
$10^{-4} \frac{m^3}{Sec}$

Fig. 9



100
90
80
70
60
50
40
30
20
10
0
● — EXPERIMENT
22 WATTS
15
9
4
$T_{MAX} - T_{\infty}$ °C
1
2
3
4
5
6
7
8
9
FLOW RATE /BOARD
10^{-4} $\frac{m^3}{Sec}$

Fig. 10

APPENDIX A

This appendix contains a listing of the FORTRAN IV digital computer program used to solve the thermal networks developed in this report. The input to the program is explained in the comment statements which precede the body of the program. Within the program the various lengths which are necessary have been taken from actual samples of the different element types and coded as numerical constants. All physical quantities are in SI units.

Following the main program is a subprogram used to calculate the function "f" defined in equation 40. Following is a series of subprograms to calculate the Bessel functions in "f". The Bessel functions are evaluated by using the series approximations found in Abromowitz and Stegun* page 378.

Also included is a listing of a typical set of input cards for one of the experimental runs.

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*Abromowitz, M., and Stegun, I. A., "Handbook of Mathematical Functions," Dover Publishing Company, New York, N.Y.

```
//BOX02MK1 JOB (0598,0918FT,59),'PROF. M. KELLEHER'
// EXEC FORTCLG,REGION.GO=100K
//FORT.SYSIN DD *
C
C
C     QFLOW = FLOW RATE PER BOARD IN CUBIC METERS PER SECOND
C
C     TINF = ENTERING AMBIENT AIR TEMPERATURE IN DEGREES KELVIN
C
C     TRINF = EFFECTIVE RADIATION SINK TEMPERATURE IN DEGREES KELVIN
C
C     XB = HORIZONTAL LENGTH OF THE BOARD IN METERS
C
C     YB = VERTICAL HEIGHT OF THE BOARD IN METERS
C
C     ZB = SPACING BETWEEN ADJACENT BOARDS IN METERS
C
C     THB = THICKNESS OF THE BOARD IN METERS
C
C     CONDB = THERMAL CONDUCTIVITY OF THE BOARD IN WATTS PER METER PER
C             DEGREE KELVIN
C
C     EB = EMISSIVITY OF THE BOARD
C
C     NEL = TOTAL NUMBER OF ELEMENTS ON THE BOARD
C
C     XE(I) = HORIZONTAL POSITION OF THE ITH ELEMENT IN METERS
C
C     YE(I) = VERTICAL POSITION OF THE ITH ELEMENT IN METERS
C
C     POWER(I):
C     IF MODE(I) = 0; THEN POWER(I) = POWER BEING GENERATED IN THE ITH
C                                     ELEMENT IN WATTS
C     IF MODE(I) = 1; THEN POWER(I) = TEMPERATURE OF THE ITH ELEMENT IN
C                                     DEGREES KELVIN
C
C     EE(I) = EMISSIVITY OF THE ITH ELEMENT
C
C     ITYPE(I) = TYPE OF ELEMENT BEING CONSIDERED WHERE:
C                ITYPE = 1  MEANS 14 PIN DIP PARALLEL TO FLOW
C                        2  MEANS 14 PIN DIP PERPENDICULAR TO FLOW
C                        3  MEANS 16 PIN DIP PARALLEL TO FLOW
C                        4  MEANS 16 PIN DIP PERPENDICULAR TO FLOW
C                        5  MEANS TO-3 CASE
C                        6  MEANS TO-66 CASE
C
C     MODE(I) = MODE OF OPERATION FOE THR ITH ELEMENT WHERE:
C               MODE = 0  MEANS ELEMENT POWER IS GIVEN AND ELEMENT
```

```
C                      TEMPERATURE IS UNKOWN
C           MODE = 1   MEANS ELEMENT TEMPERATURE IS GIVEN AND
C                      ELEMENT POWER IS UNKOWN
C
C     LEAD(I,J) = NUMBER OF LEADS FROM ELEMENT I TO ELEMENT J
C
C
      DIMENSION TYPL(6),WE(6),EL(6),TE(6),AE(6)
      DIMENSION T(50),TB(50),XE(50),YE(50),ITYPE(50),POWER(50),EE(50),
     1 MODE(50),LEAD(50,50),RBF(50),RBC(50),RBR(50),RBB(50),RL(50,50),
     2 RLINF(50),RCASE(50),RGAP(50),RMICA(50),RLS(50),TO(50)
      READ 1, QFLOW, TINF, TRINF
      PRINT 21, QFLOW, TINF, TRINF
   21 FORMAT(1H1,8HQFLOW  =,F9.6,26H   CUBIC METERS PER SECOND,10X,
     1 7HTINF  =,F7.2,8H   DEG K,10X,8HTRINF  =,F7.2,5HDEG K///)
      READ 2, XB, YB, ZB, THB, CONDB, EB, NEL
      PRINT 22, XB, YB, ZB, THB, CONDB, EB, NEL
   22 FORMAT(5H XB =,F7.4,9H   METERS,10X,4HYB =,F7.4,9H   METERS,10X,
     1 4HZB =,F7.4,9H   METERS,10X,5HTHB =,F7.4,9H   METERS,//,
     2 8H CONDB =,F7.2,20H   WATTS/METER/DEG K,10X,4HEB =,F5.2,10X,
     3 22H NUMBER OF ELEMENTS  =,I3,///)
      RNEL=FLOAT(NEL)
      DO 100 I=1,NEL
      READ 3, XE(I), YE(I), POWER(I), EE(I), ITYPE(I), MODE(I)
      IF(MODE(I).EQ.1) T(I)=POWER(I)
      IF(MODE(I).EQ.0) T(I)=POWER(I)*100.+TINF
      PRINT 23
   23 FORMAT(10X,7HELEMENT,11X,2HXE,14X,2HYE,11X,8HPOWER OR,7X,
     1 10HEMISSIVITY,8X,7HELEMENT,9X,7HMODE OF,/,10X,6HNUMBER,9X,
     2 8H(METERS),8X,8H(METERS),7X,11HTEMPERATURE,24X,4HTYPE,10X,
     3 9HOPERATION,/)
      IF(MODE(I).EQ.1) GO TO 91
      PRINT 24, I, XE(I), YE(I), POWER(I), EE(I), ITYPE(I), MODE(I)
   24 FORMAT(12X,I2,11X,F7.4,9X,F7.4,9X,F7.2,10X,F5.2,13X,I2,9X,I1,
     1 14H (POWER GIVEN)//)
      GO TO 92
   91 PRINT 25, I, XE(I), YE(I), POWER(I), EE(I), ITYPE(I), MODE(I)
   25 FORMAT(12X,I2,11X,F7.4,9X,F7.4,9X,F7.2,10X,F5.2,13X,I2,9X,I1,
     1 20H (TEMPERATURE GIVEN)//)
   92 CONTINUE
      READ 4, (LEAD(I,J), J=1,NEL)
  100 CONTINUE
      DO 95 I=1,NEL
      PRINT 26, I
   26 FORMAT(///,30H NUMBER OF LEADS FROM ELEMENT ,I2,13H TO ELEMENT J/)
      PRINT 27,(J,LEAD(I,J),J=1,NEL)
   27 FORMAT(8(3X,2HJ=,I2,2X,4HLIJ=,I2))
   95 CONTINUE
```

```
      HFREE=9.
      PI=3.14159
      SIGMA=5.6697E-08
      EPL=.85
      CONDL=386.
      XP=.0025
      TP=.0003043
      WP=.00142
      TL=.000033
      WL=.0008
      EL(1)=.0191
      EL(2)=EL(1)
      EL(3)=.0216
      EL(4)=EL(3)
      DO 90 I=1,4
      WE(I)=.0061
      TE(I)=.0030
   90 AE(I)=WE(I)*EL(I)
      EL(5)=.0222
      WE(5)=EL(5)
      TE(5)=.0089
      AE(5)=PI*EL(5)**2/4.
      EL(6)=.0157
      WE(6)=EL(6)
      TE(6)=.0075
      AE(6)=PI*EL(6)**2/4.
      VF=QFLOW/ZB/YB
      TYPL(1)=14.
      TYPL(2)=14.
      TYPL(3)=16.
      TYPL(4)=16.
      TYPL(5)=3.
      TYPL(6)=3.
      PI2=SQRT(PI)
      DO 101 I=1,NEL
  101 TB(I)=TINF+(T(I)-TINF)/2.
  102 DO 103 I=1,NEL
  103 TO(I)=T(I)
      DO 206 I=1,NEL
      IF(QFLOW) 105,105,106
  105 HB=HFREE
      GO TO 107
  106 DH=2.*ZB*YB/(ZB+YB)
      HB=3.922*SQRT(VF/XB)
  107 BMC2=2.*HB/CONDB/THB
      BMR2=8.*SIGMA*EB*TRINF**3/CONDB/THB
      BM2=BMC2+BMR2
      BM=SQRT(BMC2+BMR2)
```

```
      IT=ITYPE(I)
      RL1=SQRT(AE(IT)/PI)
      RL2=SQRT(XB*YB/PI/RNEL)
      DRBF=2.*PI*CONDB*THB*RL1*BM*(1.+((TINF-TRINF)/(TB(I)-TINF))*BMR2
     1 /BM2)
      RBF(I)=FBES(BM,RL1,RL2)/DRBF
      RBC(I)=1./HB/AE(IT)
      RBR(I)=(TB(I)-TINF)/(TB(I)-TRINF)/4./SIGMA/EB/AE(IT)/TRINF**3
      RBB(I)=RBC(I)*RBR(I)/(RBC(I)+RBR(I))
      DO 201  J=1,NEL
      RL(I,J)=0.
      IF(LEAD(I,J).EQ.0) GO TO 201
      XLIJ=ABS( XE(I)-XE(J))+ABS(YE(I)-YE(J))
      XLMC=HB*(2.*TL+WL)/CONDL/TL/WL
      XLMR=4.*SIGMA*EPL*(2.*TL+WL)*TRINF**3/CONDL/TL/WL
      XLM2=SQRT(XLMC+XLMR)
      THETR=XLMR*(TINF-TRINF)/XLM2**2
      THETI=TB(I)-TINF
      THETJ=TB(J)-TINF
      ARG=XLM2*XLIJ
      IF(ARG.GT.10.) GO TO 135
      RL(I,J)=THETI*SINH(ARG)/((THETI+THETR)*COSH(ARG)-THETJ-THETR)
     1 /CONDL/TL/WL/XLM2
      GO TO 201
  135 RL(I,J)=THETI/(THETI+THETR)/TL/WL/CONDL/XLM2
  201 CONTINUE
      RLINF(I)=THETI/CONDL/TL/WL/XLM2/(THETI+THETR)
      IF((ITYPE(I).EQ.1).OR.(ITYPE(I).EQ.3))  GO TO 202
      IF((ITYPE(I).EQ.2).OR.(ITYPE(I).EQ.4))  GO TO 203
      TOL=WE(IT)*PI2/2.
      IF(QFLOW) 120,120,121
  120 HBLAS=HFREE
      HCYL=HFREE
      GO TO 122
  121 HBLAS=3.922*SQRT(VF/TOL)
      HCYL=.0084/EL(IT)+108.59*SQRT(VF/EL(IT))
  122 RCASE(I)=(T(I)-TINF)/((HBLAS*TOL**2+HCYL*PI*EL(IT)*WE(IT))*
     1  (T(I)-TINF)+4.*SIGMA*EE(I)*(TOL**2+PI*EL(IT)*WE(IT))*
     2  TRINF**3*(T(I)-TRINF))
      GO TO 204
  203 IF(QFLOW) 125,125,126
  125 HBLAS=HFREE
      HSTAG=HFREE
      GO TO 127
  126 HBLAS=3.922*SQRT(VF/WE(IT))
      HSTAG=3.29*SQRT(VF/EL(IT))
  127 RCASE(I)=(T(I)-TINF)/((HBLAS*WE(IT)*(EL(IT)+2.*TE(IT))+HSTAG*
     1  EL(IT)*TE(IT))*(T(I)-TINF)+4.*SIGMA*EE(I)*TRINF**3*
```

```
   2 (EL(IT)*WE(IT)+2.*WE(IT)*TE(IT)+EL(IT)*TE(IT))*(T(I)-TRINF))
     GO TO 204
 202 IF(QFLOW) 130,130,131
 130 HBLAS=HFREE
     HSTAG=HFREE
     GO TO 132
 131 HBLAS=3.922*SQRT(VF/EL(IT))
     HSTAG=3.29*SQRT(VF/WE(IT))
 132 ABL=EL(IT)*(2.*WE(IT)+TE(IT))+2.*TYPL(IT)*WP*XP
     AST=2.*WE(IT)*TE(IT)
     ARAD=EL(IT)*WE(IT)+2.*WE(IT)*TE(IT)+EL(IT)*TE(IT)+TYPL(IT)*WP*XP
     RRC=HBLAS*ABL+HSTAG*AST+4.*SIGMA*EE(I)*ARAD*TRINF**3*
   1 (T(I)-TRINF)/(T(I)-TINF)
     RCASE(I)=1./RRC
 204 CONTINUE
     IF (ITYPE(I).LE.4)  GO TO 205
     RMICA(I)=.00000127/AE(IT)
     GO TO 206
 205 RGC=.04/EL(IT)/WE(IT)
     RGR=.25/SIGMA/EE(I)/EL(IT)/WE(IT)/TRINF**3
     RGAP(I)=RGC*RGR/(RGC+RGR)
     RLS(I)= XP/CONDL/WP/TP
     TAV=0.
 206 CONTINUE
     DO 213 I=1,NEL
     IF(ITYPE(I).GT.4)  GO TO 210
     RE1=1./(1./RGAP(I)+1./RLS(I))
     RRE2=1./RBF(I)+1./RBB(I)
     NL=0
     DO 211 J=1,NEL
     NL=NL+LEAD(I,J)
     IF(LEAD(I,J).EQ.0) GO TO 211
     RRE2=RRE2+FLOAT(LEAD(I,J))/RL(I,J)
 211 CONTINUE
     LINF=TYPL(I)-NL
     IF(LINF.EQ.0) GO TO 212
     RRE2=RRE2+FLOAT(LINF)/RLINF(I)
 212 RE2=1./RRE2
     RE3=1./(1./RE1+1./RE2)
     RE4=1./(1./RGAP(I)+1./RLS(I))
     RE5=1./(1./RCASE(I)+RE1/RE4/(RE1+RE2))
     IF(MODE(I).EQ.0) T(I)=RE5*POWER(I)+TINF
     IF(MODE(I).EQ.1) POWER(I)=(T(I)-TINF)/RE5
     TB(I)=RE3*T(I)/RE1+RE3*TINF/RE2
     TAV=TAV+T(I)
     GO TO 213
 210 RE1=1./(1./RBF(I)+1./RBB(I)+1./RMICA(I))
     RE2=1./(1./RBF(I)+1./RBB(I))
```

```
      RRE3=1./RCASE(I)
      NL=0
      DO 215  J=1,NEL
      NL=NL+LEAD(I,J)
  215 RRE3=RRE3+1./RL(I,J)
      LINF=IFIX(TYPL(I))-NL
      RRE3=RRE3+FLOAT(LINF)/RLINF(I)
      RE3=1./RRE3
      RE4=1./(1./RE3+RE1/RE2/RMICA(I))
      IF(MODE(I).EQ.0)  T(I)=RE4*POWER(I)+TINF
      IF(MODE(I).EQ.1)  POWER(I)=(T(I)-TINF)/RE4
      TB(I)=RE1*T(I)/RMICA(I)+RE1*TINF/RE2
  213 CONTINUE
      DO 220  I=1,NEL
      TEST=ABS(T(I)-TO(I))
      IF(TEST.GE..5   ) GO TO 102
  220 CONTINUE
      TAV=TAV/RNEL
      DELT=TAV-TINF
      PRINT 21,  QFLOW,  TINF,  TRINF
      PRINT 50
   50 FORMAT(36H FINAL  ELEMENT  TEMPERATURE  AND  POWER,///,
     1   9H   ELEMENT,/,8H   NUMBER,6X,11HTEMPERATURE,5X,5HPOWER,/)
      PRINT 51,(I,T(I),POWER(I),I=1,NEL)
   51 FORMAT(4X,I2,10X,F7.2,7X,F7.2)
      PRINT 54,  TAV,DELT
   54 FORMAT(//,23H AVERAGE TEMPERATURE   =,F7.2,10X,
     1 13HTAV  -  TINF   =,F7.2)
    1 FORMAT(F10.6,3F10.3)
    2 FORMAT(6E10.4,I5)
    3 FORMAT(2E10.4,2F10.4,2I5)
    4 FORMAT(25I3)
      END
```

```
      FUNCTION FBES(BM,X1,X2)
      B1=BM*X1
      B2=BM*X2
      FBES=(BIO(B1)*BK1(B2)+BKO(B1)*BI1(B2))/
     1            (BI1(B2)*BK1(B1)-BI1(B1)*BK1(B2))
      RETURN
      END
```

```
      FUNCTION BIO(X)
      T=X/3.75
      IF(ABS(X)-3.75) 1,1,2
    1 BIO=1.+3.5156229*T**2+3.0899424*T**4+1.2067492*T**6
     1     +0.2659732*T**8+0.0360768*T**10+0.0045813*T**12
      RETURN
    2 BIO=(0.39894228+0.01328592/T+0.00225319/T**2-0.00157565/T**3
     1     +.00916281/T**4-.02057706/T**5+.02635537/T**6-.01647633/T**7
     2     +.00392377/T**8)*EXP(X)/SQRT(X)
      RETURN
      END
```

```
      FUNCTION BI1(X)
      T=X/3.75
      IF(ABS(X)-3.75) 1,1,2
    1 BI1=(.5+.87890594*T**2+.51498869*T**4+.15084934*T**6
     1    +.02658733*T**8+.00301532*T**10+.00032411*T**12)*X
      RETURN
    2 BI1=(.39894228-.03988024/T-.00362018/T**2+.00163801/T**3
     1    -.01031555/T**4+.02282967/T**5-.02895312/T**6+.01787654/T**7
     2    -.00420059/T**8)*EXP(X)/SQRT(X)
      RETURN
      END
```

```
      FUNCTION BKO(X)
      T=X/2.
      IF(X-2.) 1,1,2
    1 BKO=-ALOG(T)*BIO(X)-.57721566+.42278420*T**2+.23069756*T**4
     1     +.0348859*T**6+.00262698*T**8+.0001075*T**10+.0000074*T**12
      RETURN
    2 BKO=(1.25331414-.07832358/T+.02189568/T**2-.01062446/T**3
     1     +.00587872/T**4-.0025154/T**5+.00053208/T**6)/SQRT(X)/EXP(X)
      RETURN
      END
```

```
      FUNCTION BK1(X)
      T=X/2.
      IF(X-2.) 1,1,2
    1 BK1=ALOG(T)*BI1(X)+(1.+.15443144*T**2-.67278579*T**4
     1-.18156897*T**6-.01919402*T**8-.00110404*T**10-.00004686*T**12)/X
      RETURN
    2 BK1=(1.25331414+.23498619/T-.03655620/T**2+.01504268/T**3
     1    -.00780353/T**4+.00325614/T**5-.00068245/T**6)/SQRT(X)/EXP(X)
      RETURN
      END
```

```
//GO.SYSIN DD *
  .00075     297.0     297.0
 .1421E+00 .1143E+00 .1524E-01 .1448E-02 .2942E+00 .8000E+00    25
 .1746E-01 .2143E-01     .5000     .85       1    0
  0  2
 .1746E-01 .4048E-01     .5000     .85       1    0
  2  0  2
 .1746E-01 .5953E-01     .5000     .85       1    0
     2  0  2
 .1746E-01 .7858E-01     .5000     .85       1    0
        2  0  2
 .1746E-01 .9763E-01     .5000     .85       1    0
           2  0  0
 .4426E-01 .2143E-01     .5000     .85       1    0
              0  0  2
 .4426E-01 .4048E-01     .5000     .85       1    0
                 2  0  2
 .4426E-01 .5953E-01     .5000     .85       1    0
                    2  0  2
 .4426E-01 .7858E-01     .5000     .85       1    0
                       2  0  2
 .4426E-01 .9763E-01     .5000     .85       1    0
                          2  0  0
 .7106E-01 .2143E-01     .5000     .85       1    0
                             0  0  2
 .7106E-01 .4048E-01     .5000     .85       1    0
                                2  0  2
 .7106E-01 .5953E-01     .5000     .85       1    0
                                   2  0  2
 .7106E-01 .7858E-01     .5000     .85       1    0
                                      2  0  2
 .7106E-01 .9763E-01     .5000     .85       1    0
                                         2  0  0
 .9786E-01 .2143E-01     .5000     .85       1    0
                                            0  0  2
 .9786E-01 .4048E-01     .5000     .85       1    0
                                               2  0  2
 .9786E-01 .5953E-01     .5000     .85       1    0
                                                  2  0  2
 .9786E-01 .7858E-01     .5000     .85       1    0
                                                     2  0  2
 .9786E-01 .9763E-01     .5000     .85       1    0
                                                        2  0  0
 .1247E+00 .2143E-01     .5000     .85       1    0
                                                           0  0  2
 .1247E+00 .4048E-01     .5000     .85       1    0
                                                              2  0  2
 .1247E+00 .5953E-01     .5000     .85       1    0
```

.1247E+00	.7858E-01	.5000	.85	1	0	2	0	2	
.1247E+00	.9763E-01	.5000	.85	1	0		2	0	2
								2	0

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